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STATIONARY TEMPERATURE MODE OF AN ANISOTROPIC THERMOELECTRIC REFRIGERATOR BASED ON THE BRIDGEMAN EFFECT

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The possibility of creating a thermoelectric refrigerator based on the Bridgeman effect is considered. The efficiency of its operation is evaluated.

In [1] a physical model by means of which the Bridgeman effect can be observed is suggested and possibilities and advantages of employment of the model for cooling are discussed in brief. We study thermoelectric cooling by means of the Bridgeman effect in more detail. Two types of refrigerator – semiannular, which has only the azimuthal component of current, and radial, where current passes along the radius – are suggested. The temperature distribution, which is taken to be one-dimensional, and the minimum temperature that can be attained by means of the refrigerators are found. The efficiency of operation is compared to the efficiency of an anisotropic thermoelectric refrigerator. Advantages of the refrigerators described in this paper over an anisotropic thermorefrigerator are noted [2].

1. Semiannular Thermoelectric Refrigerator (STR). A schematic diagram of an STR sample is given in Fig. 1a. Let the crystallographic axes of the specimen be inclined at an angle of $\pi/4$ to the x, y axes of the laboratory system of coordinates. If the kinetic coefficients of the specimen material are independent of the temperature and the coordinates and only the azimuthal component of the current $j_\varphi = j_0/r$ exists, then in the stationary mode the law of energy conservation (the generalized equation of heat conduction) in a polar system of coordinates can be written in the form

$$\frac{d^2T}{d\varphi^2} - \frac{a}{2} T (1 - \cos 2\varphi) + b = 0 , \tag{1}$$

where the temperature T is taken to be one-dimensional, i.e., dependent only on the azimuth φ; $a = \Delta\alpha j_0/\chi$; $b = \rho j_0^2/\chi$; j_0 is a constant that determines the value of the current [1].

We consider Eq. (1) along with the boundary conditions

$$T(0) = T(\pi) = T_0 . \tag{2}$$

Conditions (2) mean that the ends of the specimen are thermostated at the temperature T_0. These conditions can be easily satisfied, since the current leads should be made of metal with good electrical conductivity (e.g., copper) and good thermal conductivity; therefore if the current leads, in the thermal respect, are connected to a thermostat at the temperature T_0, then the ends mentioned will have this temperature.

We represent the solution of Eq. (1) in the form of a Fourier series in sines:

$$T(\varphi) = T_0 + \sum_{n=1}^{\infty} C_n \sin n\varphi . \tag{3}$$

Then boundary conditions (2) are satisfied automatically and we find the coefficients C_n of the Fourier series by substituting (3) in (1). Since series of the type of (3) converge rapidly, it is sufficient to restrict oneself to several terms of the sum. For three terms of expansion in (3), after substitution in (1) we find

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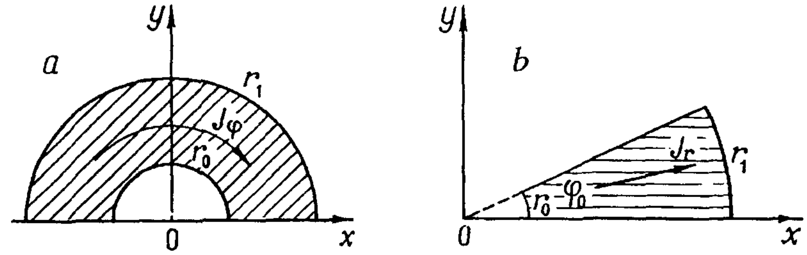

Fig. 1. Basic scheme of specimens of semiannular (a) and radial (b) thermoelectric refrigerators based on the Bridgeman effect. The direction of the principal axis of the crystal is crosshatched.

$$C_1 = \frac{(36 + 2a)\,D + aF}{a^2 + 124a + 144}\,4\,, \quad C_2 = 0\,, \quad C_3 = \frac{(4 + 3a)\,F + aD}{a^2 + 124a + 144}\,4\,,$$

where $D = (b - 0.75aT_0)E_1 + 0.25aT_0E_3$; $F = (b - 0.5aT_0)E_3 + 0.25aT_0E_1$; $E_1 = 4/\pi$; $E_3 = 4/3\pi$. We evaluate the temperature at the point $\varphi = \pi/2$:

$$T\,(\pi/2) = T_0 + 4 = \frac{(36 + a)\,D - (4 + 2a)\,F}{a^2 + 124a + 144}\,.$$

Cooling at the point $\varphi = \pi/2$ will take place when $a > 0$, $D < 0$, $F > 0$. For a numerical evaluation we take $\Delta\alpha = 10^{-4}$ V/K, $\chi = 10^{-2}$ V/(K·cm), $\rho = 10^{-3}$ Ω·cm, $j_0 = 10$ A/cm, $T_0 = 300$ K. Then $a = 0.1$, $b = 10$ K and $T(\pi/2) = 287$ K, i.e., the temperature drop is 13 K. For the same material constants and $T_0 = 300$ K a classical anisotropic thermorefrigerator operating on the transverse Peltier effect [2] gives a temperature drop of 11 K. If we compare the designs of the refrigerators mentioned, the STR is much simpler since it does not require a substrate at the temperature T_0. This substrate is characterized by the fact that it should be in isothermal contact with a lateral side of the specimen of an anisotropic thermoelectric refrigerator, and electrical contact should be absent in this case.

2. Radial Thermoelectric Refrigerator (RTR). A schematic diagram of an RTR sample is given in Fig. 1b. We consider that in this case the kinetic coefficients are also independent of the temperature and the coordinates. Then, with coincidence of the crystallographic axes and the x and y axes of the laboratory system and in the presence of just the radial component of the electric current density, i.e., $j_r = I_0/r$ and $j_\varphi = 0$, where I_0 is a constant, in a polar system of coordinates the generalized equation of heat conduction takes the form

$$r\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial \varphi^2} + \beta T\cos 2\varphi + \gamma = 0\,, \tag{4}$$

where $\beta = \Delta\alpha I_0/\chi$, $\gamma = \rho I_0^2/\chi$. Next, we assume $T = T(r)$. Then, under the condition $2\varphi << 1$, Eq. (4) can be written as

$$r\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \beta T + \gamma = 0\,. \tag{5}$$

We consider Eq. (5) along with the boundary conditions

$$T\,(r_0) = T\,(r_1) = T_0\,. \tag{6}$$

The solution of problem (5), (6) depends on the sign of β [3]. For $\beta = \nu^2 > 0$ the solution has the form

$$T(r) = \left(T_0 + \frac{\gamma}{\beta}\right) \frac{\cos\left(\frac{\nu}{2} \ln \frac{r_1 r_0}{r^2}\right)}{\cos\left(\frac{\nu}{2} \ln \frac{r_1}{r_0}\right)} - \frac{\gamma}{\beta}. \quad (7)$$

There will be no cooling here.

For $\beta = -\nu^2 < 0$ we have

$$T(r) = \left(T_0 - \frac{\gamma}{\beta}\right) \frac{r^{\nu} + (r_1 r_0)^{\nu} r^{-\nu}}{r_1^{\nu} + r_0^{\nu}} + \frac{\gamma}{\beta}.$$

For the same material constants at $I_0 = 20$ A/cm and $T_0 = 300$ K, $r_1 = 1$ cm, $r_0 = 0.1$ cm we obtain 288 K at the point $r = (r_1 - r_0)/2$, i.e., the drop is 12 K. The current strength I corresponding to this drop is $I = h\pi I_0 \varphi_0 (r_1 - r_0)$ (see Fig. 1b). The same as was said for STR can be said about the advantages of RTR compared to an anisotropic thermorefrigerator.

Thus, an anisotropic thermoelectric refrigerator operating on the transverse Peltier effect can be replaced by an STR or RTR that is simpler in design and whose working effect is the Bridgeman effect.

NOTATION

T, temperature; x, y, Cartesian coordinates; r, φ, polar coordinates; φ_0, angle of the sector; T_0, temperature of the thermostat; ρ and χ, specific electrical resistance and thermal conductivity; $\Delta\alpha$, anisotropy of the thermal emf; r_0 and r_1, inner and outer radii of the specimens; C_n, coefficients of the Fourier series; n, summation index; h, thickness of the specimen.

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